

**Time-Varying Volatility Asymmetry:  
A Conditioned HAR-RV(CJ) EGARCH-M Model**

**Ozcan Ceylan**

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## Abstract

Based on the recent developments in the high-frequency econometrics and asymmetric GARCH modeling literature, I develop a novel model that accounts for the volatility feedback and leverage effects, effectively incorporating signed continuous and jump components of the realized variance in the variance specification through an HAR forecasting model. I then condition the variance specification on the lagged realized variance and the risk aversion (that is proxied by the variance risk premium level) to analyze the eventual state-dependent variations in the volatility asymmetry. I find that the volatility asymmetry is clearly more pronounced in the periods of market stress marked by high levels of volatility and risk aversion. In addition, I reveal a further asymmetry in the asymmetric reaction patterns of the volatility to good and bad news: while the market moves through the periods of higher volatility and risk aversion, the impact of a bad news increases much more heavily than that of good news pointing to the fact that the investors become more sensible to bad news in market downturns.

**JEL Classification:** C13, C14, C32, C58, G12.

**Keywords:** Time-varying volatility asymmetry, High-frequency econometrics, EGARCH-M, HAR models, Volatility components, Variance risk premium.

## 1 Introduction

The asymmetric volatility phenomenon has long been an important issue in finance. It is now widely accepted as a stylized fact that the shocks to volatility and the shocks to returns are negatively correlated and that this correlation is more pronounced for negative return shocks<sup>1</sup>. Asymmetric volatility has important practical and theoretical implications in asset pricing, risk prediction and hedging. It implies negatively skewed return distributions and, as such, it may help to explain why severe market declines and large volatility increases tend to coincide.

In finance literature, there are two main theoretical explanations for such an asymmetric volatility reaction pattern: leverage and volatility feedback effects. According to leverage hypothesis, a negative shock to returns decreases the value of a firm's equity and hence increases the debt-to-equity ratio, rendering equity riskier and more volatile (Black (1976) and Christie (1982)). The other prominent explanation, the volatility feedback effect, involves a contemporaneous negative relationship between returns and the conditional volatility<sup>2</sup>. Given that the volatility is properly priced, an expected increase in volatility leads to an increase in the risk

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<sup>1</sup>See Engle and Ng (1993), Zakoian (1994), Bekaert and Wu (2000) and Wu (2001) among others.

<sup>2</sup>See Pindyck (1984) and French et al. (1987) for a discussion.

premium and the resulting increase in the discount rate causes an immediate price decline. The volatility feedback hypothesis may also explain the observed asymmetric nature of volatility responses to positive and negative return shocks. Irrespective of the sign of the return shock, volatility increases due to the uncertainty resolution process. Hence, as argued by Campbell and Hentschel (1992), in response to a large return shock of any sign, volatility feedback effect yields an immediate decrease in prices. As a result, the effect of an initial unexpected good news will be dampened while the effect of bad news will be amplified.

Of these two concurrent explanations of asymmetric volatility, the leverage effect is found to concern mainly the individual stocks, whereas the risk premium based explanation, the volatility feedback effect is more plausible at the market level<sup>3</sup>. Moreover, the volatility asymmetry is found to be larger for aggregate market returns than for individual stocks. Aydemir et al. (2006), Daouk and Ng (2011) and Figlewski and Wang (2001) have shown that the negative correlation between market returns and volatility is too large to be explained solely by changes in financial leverage. More importantly, they reported that the volatility asymmetry is more pronounced in down markets than in up markets.

Additional risk premium related explanations are proposed in order to reconcile these two explanations at the firm and index-levels, and at the same time, to account for the observed time-variation in the volatility asymmetry. Daouk and Ng (2011) based their explanation on the phenomenon of covariance asymmetry (i.e., covariance for stocks are higher in market downturns) that is found in Bekaert and Wu (2000), Ang and Chen (2002) and Ang et al. (2006). In Aydemir et al. (2006) time-variation in the volatility asymmetry is driven by countercyclical risk aversion which is caused by external habit formation in agents' preferences<sup>4</sup>.

This empirical evidence of time-varying asymmetry of volatility can not be captured in classical leverage-stochastic volatility models where the negative correlation between return innovations and volatility innovations remains constant throughout the sample regardless of market movements (e.g. changes in price, volatility or in risk aversion)<sup>5</sup>. Figlewski and Wang (2001) and Yu (2012) allow for changing levels of correlation between price and volatility innovations. They have shown that the effect of a positive shock to prices on variance may be negligible or even positive, whereas negative shocks will lead to an increase in variance. In Bandi and Renò (2012) the leverage parameter is linked to the variance level in the context of a continuous-time stochastic volatility model and stronger leverage effects are found to be associated with higher variance regimes.

ARCH-type models are more commonly used to examine the volatility asymmetry<sup>6</sup>. Traditional threshold

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<sup>3</sup>See, among others, Schwert (1989), Kim and Kon (1994), Andersen et al. (2001a,b), Aydemir et al. (2006).

<sup>4</sup>Time-varying risk aversion is accepted in the context of habit persistence (Campbell et al. (1997); Campbell and Cochrane, (1999)). In this consumption-based asset pricing context, countercyclical evolution of risk aversion is employed in order to explain procyclical variation of stock prices. However, since the evolution of risk aversion is related to the consumption level which is quite stable over short term, habit persistence models fail to capture severe short term fluctuations.

<sup>5</sup>Stochastic volatility model with constant leverage parameter was first estimated by Harvey and Shephard (1996).

<sup>6</sup>Time-varying volatility and volatility clustering features are modeled through the ARCH and GARCH effects in Engle (1982)

GARCH models such as Nelsons (1991) Exponential GARCH (EGARCH) and the GJR-GARCH of Glosten et al. (1993) allow one to define News Impact Functions (NIF) that measure asymmetric effects of positive and negative returns on return variance through incorporating past signed returns in the variance function. Various extensions are considered in this literature in order to capture additional observed properties of return and volatility series. Fractional integration, as proposed in FIGARCH and FIEGARCH models of Baillie et al. (1996) and Bollerslev and Mikkelsen (1996) accounts for long memory in volatility. In that vein, in-mean extensions, as first introduced by Engle et al. (1987), allow for a volatility feedback or risk-return relation effect by explicitly linking the conditional volatility to the conditional mean of returns.

The increasing availability of high-frequency data and the subsequent introduction of nonparametric daily volatility measures allowed for the development of a series of new and simple-to-implement econometric models of volatility. Based on the theory of quadratic variation, realized volatility that is computed summing finely-sampled continuously compounded intraday returns is shown to converge to an ex-post measure of integrated volatility<sup>7</sup>. This observable realized measure of volatility, when incorporated into a volatility model, results in econometric models that invariably outperform the GARCH or stochastic volatility family of models in which the volatility is a latent variable<sup>8</sup>. Among these these realized volatility-based models, the autoregressive fractionally integrated moving average (ARFIMA) model (Andersen et al. (2003)) and the heterogeneous autoregressive (HAR) models (Andersen et al., (2007); Corsi, (2009)) are capable also of capturing the observed long memory of the realized volatility series<sup>9</sup>.

Recently, several studies combine the above cited realized volatility-based forecasting models with a GARCH specification to account for time-dependent conditional heteroscedasticity in realized volatility series. Bollerslev et al. (2009a), for example, adopt the HAR model to estimate conditional variance while in the FIEGARCH-M model of Christensen et al. (2010) ARFIMA model is used instead.

In this paper, I introduce an EGARCH-type model to study the asymmetric volatility phenomenon in the French stock market using high-frequency data from January 1994 through December 2007. For the conditional variance specification, I adopt an extended version of the HAR model that explicitly accounts for continuous and jump components of realized volatility. In addition, following Barndorff-Nielsen et al. (2010), I propose a relatively new method to further decompose continuous and jump components into their positive and negative signed parts to consider eventual differences in their impacts on conditional volatility. Motivated

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and Bollerslev (1986).

<sup>7</sup>While the idea of measuring the ex-post variation of asset prices using intraday return data dates back to Merton (1980), the notion of realized variation was first formally related to the theory of quadratic variation within the context of finance by Andersen and Bollerslev (1998), Andersen et al. (2001b), Barndorff-Nielsen and Shephard (2002b) and Comte and Renault (1998).

<sup>8</sup>See, among others, Andersen et al. (2003, 2007), Corsi (2009), Corsi et al. (2008), Ghysels et al. (2006), Koopman et al. (2005), Maheu and McCurdy (2002, 2007, 2011), Martens et al. (2003).

<sup>9</sup>Formally, HAR models are not long-memory models but they can adequately reproduce the observed persistence in the sample autocorrelation functions through specifying a sum of volatility components over different horizons. In high-frequency econometrics literature, HAR models are widely used due to their estimation simplicity.

by Bollerslev et al. (2009a) I assume that the return volatility is synonymous with realized volatility when specifying the return equation and its in-mean component. The resulting HAR-RV(CJ) EGARCH-M model is enhanced by explicitly specifying the volatility of realized volatility through a GARCH process in an additional equation. This additional specification is found to capture the high persistence and clustering properties of the volatility of volatility that is documented in the literature.

To study the time-variation in the volatility asymmetry, I have conditioned the volatility equation in the model on the realized volatility levels allowing the NIFs to change according to low, high and extremely high levels of volatility. As an alternative conditioning variable, I have also calculated the daily volatility risk premia (that is considered as a good proxy of risk aversion) basing on realized and option-implied volatility series. I have found that the volatility asymmetry is more pronounced for high volatility days. Similar variation is observed through the alternative conditioning: averted investors being more sensible to market-wide return shocks, the impact of a news, and hence the volatility asymmetry will be more pronounced in periods of high risk aversion. Furthermore, I show that the impact of a bad news increases much more heavily when compared to that of good news pointing to the fact that the investors become more sensitive to bad news in market downturns.

The remainder of the paper is organized as follows. Section 2 provides a review of the relevant theories underlying the construction of the measures of realized volatility components and of variance risk premia used in the empirical model. Section 3 starts with a brief review of GARCH and HAR models and then presents the empirical model, data and results. Section 4 concludes.

## 2 Realized Variance and Variance Risk Premium Measures

### 2.1 Realized Variance, Bipower Variation and Jumps

Let  $p_t$  denote the log-price observation of a financial asset at time  $t$  and assume that  $p_t$  follows a continuous-time semimartingale jump-diffusion defined as,

$$p_t = \int_0^t \mu(s) ds + \int_0^t \sigma(s) dW(s) + \sum_{j=1}^{N(t)} \kappa(s_j) \quad (1)$$

where  $\mu(t)$  is the drift,  $\sigma(t)$  is the càdlàg instantaneous volatility,  $W(t)$  is a standard Brownian Motion,  $N(t)$  is a counting process with intensity  $\lambda(t)$  and jump size  $\kappa(s_t)$ . The corresponding increment in quadratic variation from time  $t - 1$  to  $t$  is defined as

$$QV_t = \int_{t-1}^t \sigma^2(s)ds + \sum_{t-1 < s \leq t} \kappa^2(s) \quad (2)$$

where the first component, called *integrated variance*, is from the continuous component of (1), and the second term is the contribution from discrete jumps<sup>10</sup>.

To formally define empirical variance measures, normalize the daily trading time interval to unity and denote now the intraday geometric returns for the day  $t$  as

$$r_{t,j} = p_{t-1+j/M} - p_{t-1+(j-1)/M}, \quad j = 1, 2, \dots, M \quad (3)$$

where  $M$  is the sampling frequency. Note that each period has length  $\Delta = 1/M$  and that daily open-to-close return is  $\sum_{j=1}^M r_{t,j}$ .

Barndorff-Nielsen and Shephard (2004) introduce the following estimator called (normalized) realized power variation of order  $p$  defined as

$$RPV_t(p) = \mu_p^{-1} \Delta^{1-p/2} \sum_{j=1}^M r_{t,j}^p \quad (4)$$

where  $\mu_p = E(|Z|^p) = 2^p/2 \frac{\Gamma(\frac{1}{2}(p+1))}{\Gamma(\frac{1}{2})}$ , for  $p > 0$ , where  $Z \sim N(0, 1)$ . Note that for special case of  $p = 2$  the normalizing term disappears and we have the familiar expression of the realized variance

$$RPV_t(2) = RV_t = \sum_{j=1}^M r_{t,j}^2 \quad (5)$$

that is widely discussed in literature<sup>11</sup>. It follows then by the theory of quadratic variation, for increasingly finer sampling frequency, i.e.,  $M \rightarrow \infty$  or  $\Delta \rightarrow 0$ , the realized variance converge uniformly in probability to the quadratic variation measure given in equation (2)<sup>12</sup>

$$\lim_{M \rightarrow \infty} RV_t \xrightarrow{P} QV_t = \int_{t-1}^t \sigma^2(s)ds + \sum \kappa^2(s) \quad (6)$$

In other words, the realized variance assures an an ex-post measure of the true total price variation, including the discontinuous jump part regardless of the used model or information set<sup>13</sup>.

<sup>10</sup>Note that, in the popular pure diffusive case where there is no jump in prices, the second term disappears and the quadratic variation is simply equal to the integrated variance. See, Jacod and Shiryaev (2003)

<sup>11</sup>See, for example, Andersen and Bollerslev (1998), Andersen et al. (2001b), Barndorff-Nielsen and Shephard (2002b), Comte and Renault (1998) and Meddahi (2002) among others. Following this literature we will interchangeably refer to this quantity as the realized variance or the realized volatility.

<sup>12</sup>See, Andersen et al. (2002).

<sup>13</sup>For further details on the relationship between the realized variance and the second moment of returns see Andersen et al. (2003), Barndorff-Nielsen and Shephard (2002a, 2007) and Meddahi (2003) among others.

Another estimator considered in Barndorff-Nielsen and Shephard (2004) is realized bipower variation that is defined by

$$BV_t = \mu_1^{-2} \sum_{j=2}^M |r_{t,j-1}| |r_{t,j}|, \quad (7)$$

where  $\mu_1 = \sqrt{\frac{2}{\pi}}$ .

Following Huang and Tauchen (2005), I use a slightly different notation that absorbs  $\mu_1^{-2}$  into the definition of the BV and thereby makes it directly comparable to the RV.

$$BV_t = \mu_1^{-2} \left( \frac{M}{M-1} \right) \sum_{j=2}^M |r_{t,j-1}| |r_{t,j}| \quad (8)$$

For increasingly finely sampled returns the BV measure becomes immune to jumps and consistently estimates the integrated variance in the equation (2)<sup>14</sup>:

$$\lim_{M \rightarrow \infty} BV_t \xrightarrow{P} \int_{t-1}^t \sigma^2(s) ds \quad (9)$$

Consequently, basing on the theory of quadratic variation, one can decompose the total price variation into its continuous and jump parts and isolate the jump contribution to total price variation through the difference between the nonparametric measures of RV and BV:

$$\lim_{M \rightarrow \infty} RV_t - BV_t \xrightarrow{P} \sum \kappa^2(s) \quad (10)$$

Hence, the above equation provides the basis for a nonparametric statistics for identifying jumps as emphasized by Barndorff-Nielsen and Shephard (2004, 2006). Under the assumption of no jump occurred on any particular day, Barndorff-Nielsen and Shephard (2006) first give the joint asymptotic distribution of  $RV_t$  and  $BV_t$ , conditional on the volatility path, as  $M \rightarrow \infty$

$$M^{1/2} \left[ \int_{t-1}^t \sigma^4(s) ds \right]^{-1/2} \begin{pmatrix} RV_t - \int_{t-1}^t \sigma^2(s) ds \\ BV_t - \int_{t-1}^t \sigma^2(s) ds \end{pmatrix} \xrightarrow{D} N \left( 0, \begin{bmatrix} \mathcal{V}_{qq} & \mathcal{V}_{qb} \\ \mathcal{V}_{bq} & \mathcal{V}_{bb} \end{bmatrix} \right), \quad (11)$$

where

$$\begin{bmatrix} \mathcal{V}_{qq} & \mathcal{V}_{qb} \\ \mathcal{V}_{bq} & \mathcal{V}_{bb} \end{bmatrix} = \begin{bmatrix} \mu_4 - \mu_2^2 & 2(\mu_3 \mu_1^{-1} - \mu_2) \\ 2(\mu_3 \mu_1^{-1} - \mu_2) & (\mu_1^{-4} - 1) + 2(\mu_1^{-2} - 1) \end{bmatrix},$$

and using  $\mu_1 = \sqrt{2/\pi}$ ,  $\mu_2 = 1$ ,  $\mu_3 = 2\sqrt{2/\pi}$ ,  $\mu_4 = 3$ ,

<sup>14</sup>See Barndorff-Nielsen and Shephard (2004), along with extensions in Barndorff-Nielsen et al. (2006a,b).



$$\mathcal{V}_{qq} = 2,$$

$$\mathcal{V}_{qb} = 2,$$

$$\mathcal{V}_{bb} = (\pi/2)^2 + \pi - 3.$$

The corresponding daily jump test statistic under the null of no jumps is then given by

$$\mathcal{Z}_t = \frac{RV_t - BV_t}{(\mathcal{V}_{bb} - \mathcal{V}_{qq}) \frac{1}{M} \int_{t-1}^t \sigma^4(s) ds} \quad (12)$$

where for each  $t$ ,  $\mathcal{Z}_t \xrightarrow{\mathcal{D}} N(0, 1)$  as  $M \rightarrow \infty$ . The integral in the denominator, called *the integrated quarticity*, is unobservable. To estimate it, Andersen et al. (2007) suggest using the realized Tri-Power Quarticity statistic, which is a special case of the multipower variations studied in Barndorff-Nielsen and Shephard (2004)<sup>15</sup>

$$TP_t = M \mu_{4/3}^{-3} \left( \frac{M}{M-2} \right) \sum_{j=3}^M |r_{t,j-2}|^{4/3} |r_{t,j-1}|^{4/3} |r_{t,j}|^{4/3} \quad (13)$$

where

$$\lim_{M \rightarrow \infty} TP_t \xrightarrow{P} \int_{t-1}^t \sigma^4(s) ds \quad (14)$$

The test statistic can then be employed to detect the days in which at least one statistically significant jump occurred. Let  $z_\alpha = \Phi_\alpha^{-1}$  be the  $\alpha$  quantile of a standard normal distribution. The significant jump components can be identified when the test statistic  $\mathcal{Z}$  exceeds the critical value  $z_\alpha$ .

$$J_t = (\max[0, (RV_t - BV_t)]) I_{\{\mathcal{Z}_t > \Phi_\alpha^{-1}\}} \quad (15)$$

where  $I_{\{\cdot\}}$  denotes the indicator function.

Based on this identification strategy, the continuous component of the total variation can be measured through the following switching estimator<sup>16</sup>:

$$C_t = RV_t I_{\{\mathcal{Z}_t \leq \Phi_\alpha^{-1}\}} + BV_t I_{\{\mathcal{Z}_t > \Phi_\alpha^{-1}\}} \quad (16)$$

where the truncation threshold,  $\alpha$  is considered in the range from 0.95 to 0.9999 based on the belief that jumps are rare but their sizes are large.

<sup>15</sup>One can use also the realized Quad-Power Quarticity statistic proposed by Barndorff-Nielsen and Shephard (2004) as a fair estimator of the latent integrated quarticity resulting in a slightly modified jump test statistic. See, Andersen et al. (2001a,b, 2003), and Barndorff-Nielsen and Shephard (2006) for other versions of test statistic. See also Huang and Tauchen (2005) for an overview.

<sup>16</sup>The use of this switching estimator is studied by Barndorff-Nielsen and Shephard (2004), Andersen et al. (2007) and Huang and Tauchen (2005).

### 2.1.1 Market Microstructure Noise

The jump test relies on estimates of integrated variations, which are obtained using high-frequency intraday data. The asymptotic results hinge on efficient price processes that should not be contaminated by market microstructure noise. However, a host of microstructure effects (e.g. irregular trading, discreteness of prices, bid/ask bounce etc.) introduces bias in variance estimates which becomes particularly important at very high sampling frequencies (i.e., as  $M \rightarrow \infty$ )<sup>17</sup>. One approach that is used in the literature to cope with this problem is simply to choose an intermediate sampling frequency (generally in the range of five to thirty minutes) in order to strike a reasonable balance between confounding market microstructure effects by sampling too frequently and misestimating the actual return variance when sampling too infrequently<sup>18</sup>.

Parallely, the microstructure noise may induce spurious first-order serial correlations in two adjacent intraday returns that generate an upward bias in the bipower, tri-power and quad-power estimators which are functions of adjacent returns (Andersen et al. (2007)). Clearly, this additional bias in turn will imply downward-biased jump test statistics and therefore result in errors in identifying significant jumps<sup>19</sup>. To overcome this problem, Andersen et al. (2007) suggest breaking this serial correlation by using staggered returns when computing the multipower estimators. The generalized bipower variation measure based on staggered returns is defined by

$$BV_{i,t} = \mu_1^{-2} \left( \frac{M}{M-1-i} \right) \sum_{j=2+i}^M |r_{t,j-(1+i)}| |r_{t,j}|, \quad i \geq 0, \quad (17)$$

where  $i$  denotes the offset. Similarly, the tri-power quarticity measure in the equation (13) becomes

$$TP_{i,t} = M \mu_{4/3}^{-3} \left( \frac{M}{M-2(1+i)} \right) \sum_{j=1+2(1+i)}^M |r_{t,j-2(1+i)}|^{4/3} |r_{t,j-(1+i)}|^{4/3} |r_{t,j}|^{4/3}, \quad i \geq 0. \quad (18)$$

Note that the usual multipower estimators are obtained when we apply skip-0 (i.e., when we set  $i = 0$ ). Andersen et al. (2007) propose to replace  $BV_t$  and  $TP_t$  with their staggered counterparts applying skip-1 to obtain robust jump test statistics. Following, Patton and Sheppard (2011) I use a slightly modified method instead: I construct the multipower estimators by averaging the skip-0 through skip-4 estimators.

### 2.1.2 Realized Semivariances, Signed Continuous and Jump Components

In addition to the C-J decomposition of the total quadratic variation, Barndorff-Nielsen et al. (2010) recently proposed to further decompose the realized variation and jump variation basing on the sign of intraday returns

<sup>17</sup>See, among others, Aït-Sahalia et al.(2005), Andreou and Ghysels (2002), Bandi and Russel(2006), Hansen and Lunde (2006) and Zhang et al. (2005).

<sup>18</sup>In this paper, I employed five-minutes returns which is the most commonly used sampling frequency in the literature.

<sup>19</sup>See Huang and Tauchen (2005)

to identify downside and upside risk.

$$\begin{aligned} RV_t^- &= \sum_{j=1}^M r_{t,j}^2 I_{\{r_{t,j} < 0\}} \\ RV_t^+ &= \sum_{j=1}^M r_{t,j}^2 I_{\{r_{t,j} > 0\}} \end{aligned} \quad (19)$$

Then they show that, as  $M \rightarrow \infty$ , realized semivariances converge to one-half of the integrated variance plus the sum of squared jumps with a negative/positive sign:

$$\begin{aligned} \lim_{M \rightarrow \infty} RV_t^- &\xrightarrow{P} \frac{1}{2} \int_{t-1}^t \sigma^2(s) ds + \sum \kappa^2(s) I_{\{\kappa(s) < 0\}} \\ \lim_{M \rightarrow \infty} RV_t^+ &\xrightarrow{P} \frac{1}{2} \int_{t-1}^t \sigma^2(s) ds + \sum \kappa^2(s) I_{\{\kappa(s) > 0\}} \end{aligned} \quad (20)$$

Correspondingly, the Bipower Difference estimators that may be used to estimate the magnitude of signed jumps are given as follows:

$$\begin{aligned} BPD_t^- &= RV_t^- - \frac{1}{2} \int_{t-1}^t \sigma^2(s) ds \\ BPD_t^+ &= RV_t^+ - \frac{1}{2} \int_{t-1}^t \sigma^2(s) ds \end{aligned} \quad (21)$$

To go further, I propose a novel method to decompose the continuous part of the quadratic variation into its negatively and positively signed components. To do this, I first identify significant signed jumps relying on the jump test described in section 2.1.1. Pretesting for jumps, as noted before, helps us to detect the days in which *at least* one statistically significant jump occurred. One can then simply use BPD statistics (with a non-negativity truncation) for the detected jump days to identify significant negative and positive jumps<sup>20</sup>.

$$\begin{aligned} J_t^- &= (\max[0, BPD_t^-]) I_{\{z_t > \Phi_\alpha^{-1}\}} \\ J_t^+ &= (\max[0, BPD_t^+]) I_{\{z_t > \Phi_\alpha^{-1}\}} \end{aligned} \quad (22)$$

Basing on equations (21) and (22) I define then signed continuous components for the day  $t$  as follows:

$$\begin{aligned} C_t^- &= RV_t^- I_{\{z_t \leq \Phi_\alpha^{-1}\}} + \frac{1}{2} BV_t I_{\{z_t > \Phi_\alpha^{-1}\}} \\ C_t^+ &= RV_t^+ I_{\{z_t \leq \Phi_\alpha^{-1}\}} + \frac{1}{2} BV_t I_{\{z_t > \Phi_\alpha^{-1}\}} \end{aligned} \quad (23)$$

## 2.2 Variance Risk Premium

The variance risk premium is the compensation for variance risk that stems from the randomness of return variances. Investors demand more compensation for risk when they perceive that the danger of big shocks

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<sup>20</sup>Note that, for this method, a negative and a positive jump may coexist in the same day.

to the state of economy is high <sup>21</sup>. Variance risk premium is shown to be procyclical, increasing in market downturns that are characterized by high volatility and high risk aversion. As such, it is used to capture investors' attitudes towards uncertainty (Bollerslev et al. (2011), Bakshi and Madan (2006)). Thus, it may constitute an appropriate conditioning variable for the the model that will be specified to analyze time-variations in volatility asymmetry.

Variance risk premium is defined as the difference in expected variances under risk-neutral and physical measures over the  $[t, t + n]$  time interval<sup>22</sup>.

$$VRP_t = E^{\mathbb{P}}(Var_{t,t+n} | \mathcal{F}_t) - E^{\mathbb{Q}}(Var_{t,t+n} | \mathcal{F}_t) \quad (24)$$

where  $E^{\mathbb{P}}(\cdot)$  and  $E^{\mathbb{Q}}(\cdot)$  denote the time  $t$  expectation operator under the physical and risk-neutral measures respectively. These measures can not be directly observed in practice and have to be fairly approximated.

The last term in equation 24, the risk-neutral expectation of the future variance can be computed using option prices. It can be expressed in a model-free fashion as a weighted average, or integral, of a continuum of a fixed d-maturity options <sup>23</sup>.

$$E_t^{\mathbb{Q}}(Var_{t,t+n}) = IV_{t,t+n\Delta}^* = 2 \int_0^\infty \frac{C(t+n, K) - C(t, K)}{K^2} dK \quad (25)$$

where  $C(t, K)$  denotes the price of a European call option maturing at time  $t$  with strike price  $K$ .

Methods used to construct a proxy for the physical expectation vary in practice. Carr and Wu (2008) use the ex-post forward realized variance to substitute for the expected return variance. Dreschler and Yaron (2011) use lagged implied and realized variances to forecast the expected variance. Todorov (2009) estimates the physical measure in a semi-parametric framework. Bollerslev et al. (2009b) uses a multi-frequency auto-regression with multiple lags and Zhou (2010) uses a simple auto-regression with twelve lags to estimate the objective expectation of the return variance. In this paper, following a different approach, I estimate the expected one-month-ahead variance under the physical measure through the HAR-RV(CJ) model that will be presented in detail in the following section.

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<sup>21</sup>See, for example, Bollerslev et al. (2009b), Dreschler and Yaron (2011).

<sup>22</sup>See, among others, Demeterfi et al. (1999), Britten-Jones and Neuberger (2000), Jiang and Tian (2005) and Carr and Wu (2008).

<sup>23</sup>See, Demeterfi et al. (1999) and Britten-Jones and Neuberger (2000). See also, Jiang and Tian (2005) and Carr and Wu (2008) for extensions to the case where the asset is a general jump-diffusion.

### 3 Empirical Strategy and Results

This section starts with a brief review of empirical methodology used in this study. As I specify a GARCH type model to examine the asymmetric volatility phenomenon, I leave aside the important literature on stochastic volatility models through which the volatility asymmetry can also be studied.

#### 3.1 Modeling Volatility Asymmetry : Conventional GARCH Models

The time-varying stock market volatility has been modeled as a conditional variance in the parametric Autoregressive Conditional Heteroscedasticity (ARCH) framework, as originally developed by Engle (1982). In this model, past return innovations are used to estimate the conditional variance of stock market returns.

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \epsilon_{t-i}^2, \quad \epsilon_t = z_t \sigma_t \quad (26)$$

where  $z_t \text{ iid}(0,1)$  and  $\sigma_t$  is the conditional standard deviation of return at time  $t$ , assuming that market returns follow an *AR* process.

$$r_t = \gamma_0 + \sum_{i=1}^d \gamma_i r_{t-i} + \epsilon_t \quad (27)$$

As such, this model captures some stylized facts of financial time series, such as time-varying volatility and volatility clustering.

Later, Bollerslev (1986) introduced an extension to this model, modeling variance as a function of both past innovations and past variances. The variance equation in this Generalized ARCH model is given as follows:

$$\sigma_t^2 = \omega + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 + \sum_{i=1}^q \alpha_i \epsilon_{t-i}^2 \quad (28)$$

Both models fail to capture the asymmetric volatility phenomenon. To deal with this stylized fact, many asymmetric extensions of the GARCH model have been proposed. Among the most widely spread are the Exponential GARCH (EGARCH) of Nelson (1991), the so-called GJR-GARCH of Glosten et al. (1993).

In the GJR-GARCH model the leverage coefficients of the GJR model are connected to the model through a dummy variable:

$$\sigma_t^2 = \omega + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 + \sum_{i=1}^q [\alpha_i \epsilon_{t-i}^2 + \psi_i \epsilon_{t-i}^2 I_{\{\epsilon_{t-i} < 0\}}] \quad (29)$$

where  $\psi$  captures the leverage effect. Note that for  $\psi > 0$  negative shocks have a higher impact than positive shocks of the same magnitude<sup>24</sup>. In order to assure positive variances, the model imposes positivity

<sup>24</sup>In the TGARCH model of Zakoian (1994) which is very similar to the GJR-GARCH model, conditional standard deviations are used instead of conditional variances.

constraints on parameters as  $\omega > 0$ ,  $(\alpha + \psi)$ ,  $\beta \geq 0$ .

For the EGARCH model of Nelson (1991), we do not need to impose such constraints on parameters as the variances are given in logarithmic form.

$$\ln(\sigma_t^2) = \omega + \sum_{j=1}^p \beta_j \ln(\sigma_{t-j}^2) + \sum_{i=1}^q g(z_{t-i}) \quad (30)$$

where the volatility asymmetry is captured by the NIF,  $g(\cdot)$ , which is driven by *both the magnitude and sign* of  $z_t$ ,

$$g(z_t) = \underbrace{\theta z_t}_{\text{sign effect}} + \underbrace{\phi [|z_t| - E|z_t|]}_{\text{magnitude effect}} \quad (31)$$

where  $z_t = \epsilon_t/\sigma_t$  is the normalized innovation.  $E|z_t|$  depends on the assumption made on the unconditional density<sup>25</sup>. When the standard normal distribution is assumed, we have  $E|z_t| = \sqrt{2/\pi}$ . As such, EGARCH model allows big shocks to have a greater impact on volatility, besides the sign effect that can be captured by any asymmetric GARCH model. In this model,  $\theta$  is typically negative implying that negative return shocks generate more volatility than positive return shocks, all else being equal. Combining sign and magnitude effects and holding constant the information dated  $t - 2$  and earlier we can define the News Impact Curve (NIC) to examine the relation between  $\epsilon_{t-1}$  and  $\sigma_t^2$ .

$$\begin{aligned} \sigma_t^2 &= A \exp\left[\frac{\theta+\phi}{\sigma}\epsilon_{t-1}\right], \text{ for } \epsilon_{t-1} > 0 \\ \sigma_t^2 &= A \exp\left[\frac{\theta-\phi}{\sigma}\epsilon_{t-1}\right], \text{ for } \epsilon_{t-1} < 0 \end{aligned} \quad (32)$$

where  $A \equiv \sigma^{2\beta} \exp[\omega - \phi|z_t|]$  and where  $\sigma$  is the unconditional volatility.

These GARCH models can be extended by introducing the conditional volatility into the return equation:

$$r_t = \gamma_0 + \sum_{i=1}^d \gamma_i r_{t-i} + \eta\sigma_t + \epsilon_t \quad (33)$$

where the in-mean feature,  $\eta\sigma$ , allows for the direct effect of volatility changes on asset returns (Engle et al. (1987)). In the literature, there is no agreement about the significance and the sign of the in-mean effect. In most of the asset pricing models (e.g., Sharpe (1964), Linter (1965), Merton (1973; 1980)), the presence of a positive equity premium (or risk-return tradeoff) implies a positive volatility-return relation. On the other hand, the in-mean effect may also be driven by the volatility feedback which induces a negative coefficient for this feature. To sum up, both the risk-return tradeoff and the volatility feedback may be reflected by the

<sup>25</sup>In the GARCH literature, the most used density distributions, besides the normal distribution, are the  $t$ -distribution and the Generalized Error Distribution which may allow for fat tails.

in-mean effect and the sign of the in-mean coefficient is determined depending on the relative importance of these two opposing factors.

### 3.2 Forecasting Variance: HAR Models

Several empirical studies point to the long-memory dependence in market variance. ARFIMA models are widely used to capture this long-memory property in realized variances<sup>26</sup>. Another option, that is much simpler to implement, is to use a Heterogeneous Autoregressive (HAR) model to reproduce slowly decaying autocorrelation structure found in long-memory models.

The HAR model proposed by Corsi (2009) is a straightforward unfolding of Heterogeneous ARCH (HARCH) models analyzed earlier in Muller et al. (1997). An HAR model can be specified as a multi-component variance model in which the conditional variance is parametrized as a sum of variance components over different horizons. Generally, in its simplest form, an HAR-RV model is estimated through the the sum of daily, weekly and monthly variances

$$RV_t = \beta_0 + \beta_D RV_{t-1} + \beta_W RV_{t-5:t-1} + \beta_M RV_{t-22:t-1} \quad (34)$$

where

$$RV_{t+1-k:t} = \frac{1}{k} \sum_{j=1}^k RV_{t-j} \quad (35)$$

and where the coefficients  $\beta_0$ ,  $\beta_D$ ,  $\beta_W$ ,  $\beta_M$  are determined through an OLS estimation<sup>27</sup>.

Taking in account the possibly separate informative content of jumps in volatility forecasting, Andersen et al. (2007) propose to extend the above model including a jump component. The resulting HAR-RV-J model is specified as

$$RV_t = \beta_0 + \beta_D RV_{t-1} + \beta_{JD} J_{t-1} + \beta_W RV_{t-5:t-1} + \beta_M RV_{t-22:t-1} \quad (36)$$

Parallely, one can specify an HAR-RV-CJ model through the C-J decomposition discussed above.

$$RV_t = \beta_0 + \beta_{CD} C_{t-1} + \beta_{JD} J_{t-1} + \beta_W RV_{t-5:t-1} + \beta_M RV_{t-22:t-1} \quad (37)$$

Alternatively, the semivariance decomposition may be applied to increase the forecasting performance.

<sup>26</sup>See, for example, Andersen et al. (2003), Areal and Taylor (2002), Koopman et al. (2005), Pong et al. (2004).

<sup>27</sup>For logarithmic versions, variance components are defined as  $\log(RV_{t+1-k:t}) = \frac{1}{k} \sum_{j=1}^k \log(RV_{t-j})$ .

$$RV_t = \beta_0 + \beta_D^+ RV_{t-1}^+ + \beta_D^- RV_{t-1}^- + \beta_W RV_{t-5:t-1} + \beta_M RV_{t-22:t-1} \quad (38)$$

Combining all and basing on C-J decomposition of realized semivariances discussed in the previous section, I specify the new HAR-RV(CJ) model as

$$RV_t = \beta_0 + \beta_{CD}^+ C_{t-1}^+ + \beta_{CD}^- C_{t-1}^- + \beta_{JD} J_{t-1} + \beta_W RV_{t-5:t-1} + \beta_M RV_{t-22:t-1} \quad (39)$$

This last model constitute the basis for the variance equation of the HAR-RV(CJ) EGARCH-M specification that will be presented in the following subsection. The corresponding log version of the HAR-RV(CJ) model has thus to be expressed as follows:

$$\begin{aligned} \log(RV_t) = & \beta_0 + \beta_{CD}^+ \log(C_{t-1}^+) + \beta_{CD}^- \log(C_{t-1}^-) + \beta_{JD} \log(J_{t-1} + 1) \\ & + \beta_W \log(RV_{t-5:t-1}) + \beta_M \log(RV_{t-22:t-1}) \end{aligned} \quad (40)$$

### 3.3 HAR-RV(CJ) EGARCH-M Model

In the high-frequency econometrics literature presented in the previous section, it has been shown that under empirically realistic conditions the conditional expectation of the integrated variance is equal to the conditional variance of returns and that the latent integrated variance can be approximated fairly well by the realized variance. It has been also shown that this observable realized measure of volatility, when incorporated into a volatility model, results in econometric models that invariably outperform the GARCH or stochastic volatility family of models in which the volatility is a latent variable (cf. footnote 8). Hence, following Bollerslev et al. (2009a), I assume that the return volatility is synonymous with realized volatility,  $\sigma_t = \sqrt{RV_t}$ , to redefine the equation (33) as<sup>28</sup>

$$r_t = \gamma_0 + \sum_{i=1}^d \gamma_i r_{t-i} + \eta \sqrt{RV_t} + \epsilon_t \quad (41)$$

where  $\epsilon_t$  is now given as  $\sqrt{RV_t} z_t$ .

To specify the conditional variance equation I rely on the logarithmic HAR-RV(CJ) model given in the equation (40). I then incorporate the NIF presented in the equation (31) in order to allow for the volatility asymmetry in this equation. Further, to accommodate the observed volatility clustering in realized volatility, I extend the conditional variance specification through a separate GARCH-type specification for the variance of variance as in Corsi et al. (2008)<sup>29</sup>. The resulting variance specification is thus expressed as follows:

<sup>28</sup>See also Maheu and McCurdy (2011) for an alternative method to incorporate realized measures into GARCH models.

<sup>29</sup>See also Bollerslev et al. (2009a) for a similar extension



$$\begin{aligned}\log(RV_t) = & \beta_0 + \beta_{CD}^+ \log(C_{t-1}^+) + \beta_{CD}^- \log(C_{t-1}^-) + \beta_{JD} \log(J_{t-1} + 1) + \beta_W \log(RV_{t-5:t-1}) \\ & + \beta_M \log(RV_{t-22:t-1}) + \theta z_{t-1} + \phi [|z_{t-1}| - E|z|] + \sqrt{h_t} u_t\end{aligned}\quad (42)$$

$$h_t = \nu_0 + \sum_{i=1}^m \nu_i h_{t-i} + \sum_{j=1}^s \zeta_j u_{t-j}^2$$

where  $h_t$  denotes the variance of variance.

Motivated by Bandi and Renò (2012), I apply then some conditioning variables on the variance equation to analyze the eventual time-variation in the volatility asymmetry. Precisely, I condition the NIF on non-overlapping intervals of the variance and of the variance risk premium levels. To illustrate, take the case where the realized variance is used as conditioning variable and reformulate the variance equation as

$$\log(RV_t) = (HAR - RV(CJ))_{t-1} + g(z_{t-1}) I_{\{q_x \leq RV_{t-1} < q_y\}} + \sqrt{h_t} u_t \quad (43)$$

where  $q_x$  refers to  $x$ th quantile of the conditioning variable (i.e the realized variance for this illustration). Note that for each interval, the estimated parameters in the NIF are denoted now as  $\theta_{y-x}$  and  $\phi_{y-x}$  and that  $z_t$  may be expressed explicitly as

$$(z_t)_{y-x} = \left( \frac{r_t - \gamma_0 - \sum_{i=1}^d \gamma_i r_{t-i} - \eta \sqrt{RV_t}}{\sqrt{RV_t}} \right) I_{\{q_x \leq RV_{t-1} < q_y\}} \quad (44)$$

and used to calculate the interval-mean of the standardized residuals  $E(|(z_t)_{y-x}|)$ .

Constrained by the sample size, the number of intervals is limited to three. For each of the two conditioning variables, the quantile-intervals are set to  $(x-y) = (0-50), (50-90)$  and  $(90-100)$  corresponding respectively to the periods of low, high and extreme market volatility (resp. risk aversion) in the case where the realized variance (resp. variance risk premium) is employed as the state variable.

After all, NIFs are rescaled in order to infer the "implied" ones. The scaling factor  $\widehat{S}$  is calculated as

$$\widehat{S} = \frac{\widehat{std}(g(z_{t-1}) I_{\{q_x \leq RV_{t-1} < q_y\}})}{\widehat{std}([\log(RV_t) - (HAR - RV(CJ))_{t-1}] I_{\{q_x \leq RV_{t-1} < q_y\}})} \quad (45)$$

### 3.4 Data and Summary Statistics

In the empirical study, the daily realized measures are computed basing on high-frequency data for the French CAC40 index, ranging from January 1994 to December 2007. As discussed in 2.1.1., I employed five-minutes sampling to calculate intraday index returns. Trading on the CAC40 starts at 9:00am and continues till 5:30pm. A typical day has thus  $22 \times 102 = 2246$  five-minutes subintervals. I exclude all overnight returns.

The significant jumps identified setting the truncation threshold,  $\alpha$ , to 0,999.

The high-frequency data for the CAC40 index are provided by the Euronext until the end of 2004. For the rest, NYSE Euronext provides only the tick-by-tick transaction prices for all securities. I employed a slightly modified version of the tick method of Dacorogna et al. (2001) to pick out the security prices at each intraday interval. These prices are then used to construct the CAC40 index values.<sup>30</sup>

As mentioned in 2.2., the variance risk premium series that I employ as a conditioning variable in the model is based on the risk-neutral and objective measures of future index variance. For the risk-neutral measure I used the squared VCAC index that is directly available from the NYSE Euronext since January 2000. The VCAC index is based on the CAC40 index options covering the out-of-the-money strike prices for the near and next term maturities and constructed following the widely used VIX methodology of the Chicago Board of Options Exchange<sup>31</sup>. For the period from January 1994 to December 1999 that is not covered by the VCAC index, I rely on high-frequency index options prices with one month to maturity, which are also provided by the Euronext, to compute the implied volatility series to be merged with the VCAC index. The expected one-month-ahead variance under the physical measure is proxied by  $22 \times RV_{t+22:t+1}$  estimated through the HAR-RV(CJ) model given in the equation (39)<sup>32</sup>.

Summary statistics for the realized measures used in the estimations are reported in Table 1. Moment characteristics of return and realized variance series for the French stock market are in accordance with the empirical finance literature: leptokurtic unconditional distributions with a negative skew for the returns and highly positive skew for the volatility measures. Logarithmic transformation renders all the realized volatility measures approximately normal. The leptokurtosis in negatively signed semivariances and continuous variance components are significantly more pronounced than in their positively signed counterparts. The mean, median and maximum values of realized (semi)variance exceed those of the (signed) continuous variance components.

Details concerning the daily estimates of the variance risk premium are given in Table 2. To represent the level of risk aversion, the variance risk premium series are multiplied by  $-1$ . Unconditional distributions of all the three series are positively skewed and fat-tailed. The mean and the median of the risk-neutral expectation of future variance are far superior to those of the expected variance under physical measure, pointing to negative premium for the variance risk (corr. risk aversion). As expected, all of the three series exhibit slowly decaying autocorrelation functions.

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<sup>30</sup>Beginning from December 1st, 2003, index is calculated on the basis of the of the free-float capitalization instead of the market capitalization. The details for this new calculation method can be found on [www.euronext.com](http://www.euronext.com). The list of the CAC40 index components as well as their number of shares and their free float and the capping factor which is taken into account in the calculation are periodically updated and published on the website.

<sup>31</sup>This method is developed by Demeterfi et al. (1999).

<sup>32</sup>Daily-standardized estimates of the variance risk premium may be obtained by dividing the monthly estimates by 22.

### 3.5 Results

For each of the individual equations, lag structures are determined basing on the AIC and BIC statistics of the overall model while ensuring that the individual parameters are all significant (at the 5% level) based on their individual  $t$ -test statistics. In the preferred HAR-RV(CJ) EGARCH-M model, the conditional return equation is specified as a first order autoregressive process augmented by a statistically significant in-mean component while the variance of variance is modeled through a GARCH (1,1) process.

Estimation results concerning the preferred model and its conditioned versions are reported in Table 3. The negative autoregression parameter for returns captures the mean-reversion property of the return series. The estimated negative parameter for the in-mean component indicates that the volatility feedback effect dominates the risk-return tradeoff in determining the contemporaneous volatility-return relation. Parameter estimates concerning the variance of variance are in line with the previous studies in the literature<sup>33</sup>: in all estimations, we have  $(\nu_1 + \zeta_1) \approx 0.9$  pointing to high persistence in the variance of variance. The daily, weekly and monthly volatility components are all highly significant and their estimates are consistent with the earlier results in the literature for the HAR-RV model.

Turning to volatility asymmetry (or leverage effect) parameters in the conditional variance equations, the highly statistically significant estimates for  $\theta$  and  $\phi$  reflect the importance of asymmetry in volatility. Particularly, the negative estimates for the sign effects in all models imply that a negative return shock generates a much larger increase in the volatility than does a positive shock of the same magnitude. When we focus on the model in which the lagged RV is used as a conditioning variable we see a clear asymmetry in the volatility asymmetry: the strength of sign and magnitude effects increases with the volatility level. We have a similar asymmetric pattern of volatility asymmetry when we use a proxy of risk aversion level as conditioning variable. For these two conditioned models, the news impact curve parameters corresponding to each quantile-interval of conditioning variables are given in Table 4. The impact of a standardized return innovation of any sign increases with the variance level and, to a lesser extent, with the level of the risk aversion<sup>34</sup>. Moreover, the increases in the impact of a bad news are larger than those of a good news: given that the NIFs are *exponential* functions of volatility asymmetry parameters,  $\theta$  and  $\phi$ , the ratio of the impact of a bad news to that of a good news for each quantile-interval will be an exponential function of  $(-2\theta)$  and will thus also increase in the periods of high volatility and of high risk aversion. Especially in the periods of extreme volatility and of extreme risk aversion, this disproportionality in the volatility asymmetries induced by bad news and good news becomes much more remarkable.

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<sup>33</sup>See, for example, Corsi et al. (2008) and Bollerslev et al. (2009a)

<sup>34</sup>This difference in the NIC patterns between these two models is partly due to disproportionalities in the ratios between interval-based standard deviations of NIFs and of variance equation errors. From Table 5, it can be seen that this difference in NIC patterns between the conditioned models is attenuated when we apply the scaling factors given in the equation (45) to the corresponding NIFs taking in account these disproportionalities.

## 4 Conclusion

Combining the most recent developments in the high-frequency econometrics and asymmetric GARCH modeling literature I develop a novel model that accounts for the volatility feedback and leverage effects, effectively incorporating signed continuous and jump components of the realized variance in the return variance specification through an HAR forecasting model. Motivated by the recent empirical results on the time-variation in the volatility asymmetry, I then condition the variance specification on the lagged realized variance and the risk aversion (that is proxied by the variance risk premium levels) to analyze the eventual changes in the asymmetric relation between (signed) return and volatility innovations in different states of the French market.

Through the estimation of the accurately specified non-conditioned HAR-RV(CJ) EGARCH-M model, I first show that there are significant asymmetries in the return-volatility relation. Then, conditioning this model on lagged state variables, I find that this asymmetric relationship itself varies asymmetrically depending on different states of the market: the volatility asymmetry is clearly more pronounced in the periods of market stress marked by high volatility and risk aversion. In addition, estimation results introduce a further asymmetry in the asymmetric reaction patterns of the volatility to good and bad news: while the market moves through the periods of higher volatility and risk aversion the impact of a bad news increases much more heavily when compared to that of good news pointing to the fact that the investors become more sensible to bad news in market downturns.

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Table 1: Descriptive Statistics for Realized Measures

Series	Mean	Median	Std. Dev.	Skewness	Exc. Kurt.	min	max
$r_t$	-0.00546	0.01307	1.13293	-0.08535	4.10028	-8.31999	7.28247
$RV_t$	1.09325	0.69435	1.71428	10.97925	208.85237	0.03717	46.69333
$\sqrt{RV_t}$	0.92697	0.83328	0.48379	2.69087	16.38033	0.19281	6.83325
$RV^+$	0.53378	0.33608	0.82092	10.47967	199.50249	0.01611	21.65783
$RV^-$	0.55947	0.321	1.03612	12.67329	247.27286	0.01362	25.0355
$C$	1.00476	0.66233	1.34375	8.6443	135.71216	0.03718	29.573
$C^+$	0.49028	0.3227	0.65344	9.03016	170.88969	0.01611	17.806
$C^-$	0.51045	0.30717	0.765	11.18232	246.61645	0.01362	22.98
$\log(RV_t)$	-0.36687	-0.36478	0.90949	0.22557	0.30362	-3.29	3.84
$\log(RV_t^+)$	-1.09474	1.09041	0.92659	0.19321	0.25136	-4.129	3.075
$\log(RV_t^-)$	-1.12111	-1.13631	0.99225	0.17714	0.25355	-4.301	3.22
$\log(C^+)$	-1.14296	-1.13103	0.90749	0.10897	0.07317	-4.129	2.88
$\log(C^-)$	-1.16121	-1.18016	0.96988	0.09319	-0.00001	-4.301	3.13
$\log(RV_{t-5:t-1})$	-0.36545	-0.35562	0.81929	0.16289	0.01114	-3.02	2.56
$\log(RV_{t-22:t-1})$	-0.36529	-0.32128	0.76858	0.0901	-0.41333	-2.24	1.82

Table 2: Variance Risk Premium

The expected variance under the  $\mathbb{Q}$  measure is computed as  $E_{t,t+1}^{\mathbb{Q}} = (VCAC)^2/252$ . One day ahead expected variance under the physical measure  $E_{t,t+1}^{\mathbb{P}}$  is approximated by  $RV_{t+22:t+1}$  estimated through an HAR-RV(CJ) model as

$$RV_{t+22:t+1} = 0.4597 + 0.2377C_{t-1}^- + 0.0916C_{t-1}^- - 0.0352J_{t-1} + 0.2356RV_{t-5:t-1} + 0.1941RV_{t-22:t-1}.$$

Series	Mean	Median	Std.Dev.	Skewness	Exc.Kurt.	$\rho_1$	$\rho_2$	$\rho_4$	$\rho_7$	$\rho_{10}$	$\rho_{20}$
$E^{\mathbb{P}}$	1.0937	0.9133	0.6619	4.2062	29.7696	0.926	0.887	0.847	0.761	0.693	0.493
$E^{\mathbb{Q}}$	2.2343	1.6881	1.9543	3.0672	12.9078	0.94	0.915	0.896	0.856	0.844	0.747
-VRP	1.1446	0.7008	1.6298	3.1352	14.3442	0.903	0.866	0.85	0.802	0.797	0.701

Table 3: HAR-RV(CJ) EGARCH-M Model Estimation Results

Parameter	Standard Model		Conditioning on Lagged RV		Conditioning on Lagged -VRP	
	Estimate	t-stat	Estimate	t-stat	Estimate	t-stat
$\gamma_0$	0.1596	4.81	0.1707	5.10	0.1655	4.95
$\gamma_1$	-0.0462	-3.01	-0.0502	-3.28	-0.0492	-3.2
$\eta$	-0.1756	-4.4	-0.1864	-4.67	-0.1808	-4.52
$\beta_0$	0.1354	6.71	0.1616	7.96	0.1438	7.11
$\beta_{CD}^-$	0.1386	6.02	0.1442	6.24	0.1439	6.15
$\beta_{CD}^+$	0.1491	6.03	0.1638	6.61	0.147	5.88
$\beta_{JD}$	0.1502	2.82	0.2121	3.9	0.1494	2.77
$\beta_W$	0.4194	12.75	0.4192	12.82	0.4247	12.88
$\beta_M$	0.2402	9.02	0.2339	8.85	0.2413	9.04
$\theta$	-0.053	-4.98				
$\phi$	0.0847	7.32				
$\theta_{100-90}$			-0.121	-5.36	-0.0818	-4.54
$\phi_{100-90}$			0.2369	7.18	0.1481	5.37
$\theta_{90-50}$			-0.0644	-4.54	-0.0494	-3.66
$\phi_{90-50}$			0.0889	4.76	0.0741	4.16
$\theta_{50-0}$			-0.0351	-2.82	-0.0421	-2.94
$\phi_{50-0}$			0.0501	3.14	0.0511	2.8
$\nu_0$	0.0188	2.53	0.0222	2.95	0.0225	2.97
$\nu_1$	0.0481	3.42	0.0518	3.82	0.0533	3.87
$\zeta_1$	0.8722	20.09	0.8529	19.79	0.8511	19.73
Log Likelihood		-7944		-7921		-7938

Table 4: News Impact Curve Parameters

All parameter values are multiplied by 100 for presentation issues.

Conditioning Variable	$(\phi + \theta)_{100-90}$	$(\phi - \theta)_{100-90}$	$(\phi + \theta)_{90-50}$	$(\phi - \theta)_{90-50}$	$(\phi + \theta)_{50-0}$	$(\phi - \theta)_{50-0}$
Lagged RV	11.5926	35.7886	2.4529	15.3349	1.4962	8.5202
Lagged -VRP	6.6274	22.9874	2.4702	12.3522	0.8991	9.3231

Table 5: Implied News Impact Curve Parameters

All parameter values are multiplied by 100 for presentation issues.

Conditioning Variable	$(\phi + \theta)_{100-90}$	$(\phi - \theta)_{100-90}$	$(\phi + \theta)_{90-50}$	$(\phi - \theta)_{90-50}$	$(\phi + \theta)_{50-0}$	$(\phi - \theta)_{50-0}$
Lagged RV	5.5308	17.0747	0.4832	3.021	0.1206	0.6867
Lagged -VRP	2.7032	9.3761	0.3736	1.8811	0.0733	0.7606