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Abstract

Unrecorded economic activities have an important weight, especially in developing countries where environmental regulations are gradually pursued. Both theoretical and empirical studies on the subject which do not take into account the existence of unrecorded economy may not provide a complete insight on the effects of both fiscal and environmental enforcement policies. After a brief review of the relevant literature, this paper develops an economic model to analyze the potential impacts of environmental regulatory policies on the size of unrecorded economy. Two different cases are considered: first, firms' emissions and productions are audited with exogenous probabilities which may be different from each other; second, a unique probability-to-audit function is determined to audit both emissions and productions of firms whether in recorded or unrecorded economy. The form of this function is specified using the cointegration technique. The results in this paper essentially show that environmental regulations may increase the size of unrecorded economy. The paper also attempts to give a precise limit value for the environmental tax rate exceeding which may induce a rise in the extent of unrecorded activities.

Keywords: Environmental taxation; Unrecorded economy; Duopolistic competition

JEL classification: D43; H32; Q58

1 Introduction and relation to previous literature

To what extend does the economic theory of environmental regulation explain the unfolding of firms' behavior and would it be adequate to apply it on an "as-is" basis to both developed and developing countries? In order to provide a sufficiently well-developed response to this question one should take into account the weight of unrecorded activities in the overall economy, particularly in developing countries. Since in these countries the size of unrecorded economy is estimated to be very large (see Table 1) the overall impact of environmental regulation should be re-examined both theoretically and empirically. Hence, our primary interest in the problem studied here arose from the fact that the results from the existing literature may not be reliable; thus, for the case for most of the developing countries, attempts to give recommendations and policy implications following previous studies on environmental regulation may not go further than being inadequate and even misleading.

Several papers investigate only unrecorded economy, its causes and consequences: unemployment, increased regulation in the recorded economy, corruption, rise of the tax burden are the most cited causes of unrecorded economy while existence of a Laffer curve[1], reduced effectiveness of macroeconomic policies, economic instability, distortions in resource allocations and underinvestment represent its main consequences (Schneider and Enste, 2000; Eilat and Zinnas, 2002).[2] In these studies, special atten-

[1] This inverted U shaped curve shows that governments may increase their tax revenues by increasing the tax rate up to an *optimal* tax rate beyond which further increase of taxation decreases tax revenues. In the presence of an unrecorded economy the tax base is smaller than it should be without unrecorded economy. Increasing taxes to compensate the revenue loss resulting from unrecorded activities drives firms out of the official economy, thus increasing further the size of unrecorded economy. This vicious circle characterizes at the same time cause and consequence of the unrecorded economy.

[2] One of the most used definitions of unrecorded economy is from Smith (1994, p.18) who defines it as "market-based production of legal goods and services that escapes detection in the official estimates of GDP due to the efforts of some businesses and households to keep their activities undetected". To

tion is given, on the one hand, to the methodological issues in the estimation of the size of unrecorded economy and on the other hand, to the overall macro- and micro-economic impacts of unrecorded activities without providing necessary and useful implications for the environmental policy more specifically for the possible relationship between environmental regulation and the size of unrecorded economy.

[Table 1 here]

Other studies concentrate only on the environmental regulation and enforcement policy without examining whether there exists an unrecorded economy. Since the pioneering study of Pigou (1920) it has been recognized that a regulatory authority can internalize external costs resulting from production (emissions) by introducing an environmental tax determined by the marginal damage created from this activity (i.e. Pigouvian tax). Obviously, the world is not as simple as Pigou's (1920) basic economic model. The main problem in this area is that it is not easy or cheap to identify the emission level of each firm, therefore an efficient enforcement mechanism is needed in order to minimize the total social loss (Becker, 1968). Following this, more recent studies addressed monitoring and optimal enforcement mechanism design issues and reported several interesting findings (see Cohen (1999) and Lewis (1996) for a survey). In the same line of research, for example, in an oligopolistic competition framework, Damania (2000) points out that a high emission tax rate may not be effective in decreasing total emissions and in some circumstances it may even increase them. On the other hand, Macho-Stadler and Pérez-Castrillo (2006) argue that in order to decrease

conserve space, we do not discuss in detail definition and theoretical and empirical foundations of the estimation of unrecorded economy which are well documented in the literature. For a good overview of these and other issues discussed in this paragraph see for example Feige (1990) and also Karanfil and Ozkaya (2007).

total emissions the most suitable strategy that can be adopted by the environmental enforcement agency is a "discriminatory" audit strategy which consists of focusing on both the "easier-to-detect" firms and firms that value pollution less. Furthermore, some other recent studies on the relationship between enforcement mechanism and firm's compliance behavior demonstrated that in a market involving widespread non-compliant firms environmental quality (lower emissions) is positively associated with managers' risk aversion (Stranlund, 2008) and that an increase in enforcement efforts may provide better environmental results inducing not only non-compliant firms to comply with the regulation but also *over*-compliant firms to reduce further their emissions (Shimshack and Ward, 2008).

None of the aforementioned studies has assessed whether taking into account unrecorded economy leads to a substantial change in the conclusions reached. To date, the only study available addressing the issue of variability of the results obtained in an energy-environment-income nexus once unrecorded activities are accounted for is that of Karanfil (2008) who concluded that for the case of Turkey there exists a long-run causality from *official* GDP to energy consumption while *true* GDP and energy consumption are found to be neutral with respect to each other and that as a results, adjustment policies and structural reforms aiming at decreasing the size of unrecorded economy may not serve as a complement to environmental policies which may be feasible without harming *recorded* economic growth.

In consequence of the above-mentioned facts, for developing countries, the analysis conducted in this paper is much more appropriate than earlier papers in this field in at least two ways: first it considers an economy composed of both recorded and unrecorded activities; second, the impacts of an environmental enforcement policy on the size of unrecorded economy are analyzed, which, to the best of our knowledge, has not done before.

The outline of the paper is the following: In Section 2, the model environment is described and the assumptions on which the model is based are discussed. In Section 3, behaviors of firms subject to non-cooperative fiscal and environmental regulations are analyzed and after determining reaction functions which give the firms' Cournot equilibrium quantities, some stability and comparative static analysis are conducted. Moreover, the results from Cournot game are compared to those obtained in Stackelberg market. Proposing another enforcement mechanism where firms are audited on their productions and emissions with a unique probability which is supposed to be a function of the reported production, Section 4 establishes a threshold rate of environmental tax which, if it is exceeded, may lead to an increase of the size of unrecorded economy. To provide further information, a similar Stackelberg framework is used as in Section 3. The final section concludes the paper and discusses in brief detail the implications of the findings.

2 Model environment

We model an industry where there are both recorded and unrecorded economic activities. We deal with two representative duopolistically competitive firms. Existence of duopolistic competition in the presence of unrecorded economy may be perceived in the following way: In an industry we may have a large number of differentiated goods and in the production of some homogeneous goods there may be a duopolistic competition. Thus the industry would be composed of a large number of duopolistically competitive firms for every of these homogeneous goods. In this situation the regulatory authorities can perfectly observe neither the production nor the emission level of each firm. As a result, they use auditing mechanisms to create incentives for truthful revelation.[3]

Each firm faces a linear market demand for its homogeneous product; q^R and q^U (Superscripts on a variable or on a parameter denote activity characteristic of the firm throughout the paper; R stands for recorded economy and U for unrecorded economy). The homogeneity assumption is not unrealistic since unrecorded economy is generated by mainly tax evasion in economic activities like peddling or hawking where the product differentiation is not very great (Karanfil, 2008).

On the other hand, let the linear inverse market demand function be $p(Q) = a - bQ$ where $p(Q)$ and b stand for market price and the slope of the demand function respectively. Q is the aggregate output, that is, $Q = q^R + q^U$.

To make ideas more concrete and to have simpler and analytically more tractable model we shall also make the following assumptions. As the constraints faced by firms in the presence of unrecorded economy are asymmetric, an asymmetric cost function should be used for each firm. Consider the following cost function of a representative firm reporting all its activity (henceforth firm R): $c^R(q^R, x^R) = \varphi^R \frac{1}{2} q^{R^2} + \phi^R \frac{1}{2} x^{R^2}$. Marginal cost of production and marginal cost of pollution abatement effort x^R are determined by the production efficiency φ^R and the abatement efficiency ϕ^R respectively. The polluting emission level of the firm R, e^R, is given by a linear function of q^R and x^R. More formally, let the emission coefficient be denoted by δ^R, we have

[3]Besides these properties, it is evident that the duopoly setup provides a simple and transparent framework to study the above research issues. However, other market structures can of course be considered in this field. For example, Fujiwara (2009) discusses the effects of environmental policies in a model of polluting oligopoly with product differentiation.

$e^R = \delta^R q^R - x^R$.

The other representative firm producing in the unrecorded economy (henceforth firm U) neither reports any of its income and its polluting emissions nor performs abatement. Thus the cost function for the firm U reduces to $c^U(q^U) = \varphi^U \frac{1}{2} {q^U}^2$ and its emission level is defined simply by $e^U = \delta^U q^U$.[4]

Once we have described our specification of the firms' behaviors in both recorded and unrecorded economic activities, the crucial feature is how strategies are affected by both environmental and income tax enforcement policies. The next sections will address this question considering different cases with respect to the existence or non-existence of *environmental-fiscal enforcement cooperation.*

3 Non-cooperative policy

In the first framework we develop a model in which there is no cooperation between environmental and finance regulatory authorities. It means that the economic (more specifically fiscal and finance) policies to combat unrecorded economy and the environmental policies to decrease pollutant emissions are not coordinated. Hence, the information which can be used to design an enforcement policy is not common to all the enforcement agencies. As a result, firms' emission levels and productions are audited with different exogenous probabilities. The next section then introduces another enforcement policy mechanism in a cooperative policy scheme.

In the recorded economy, where there is no tax evasion, the firm R decides how

[4]As it will be shown below, although the firm U does not engage in pollution abatement, both the firms R and U are subject to environmental regulation and enforcement. Besides, this makes the firm U have a "total" risk-seeking behavior.

much to produce (q^R), to give report on the emissions (z) and to invest in abatement technologies (x^R) solving the following maximization problem:[5]

$$\begin{aligned} Max\Pi^R &= [p(q^R+q^U)-t_Y]q^R - c^R(q^R,x^R) - t_E z \\ &\quad -\alpha_R[t_E(e^R(q^R,x^R)-z)+\theta(e^R(q^R,x^R)-z)] \end{aligned} \tag{1}$$

where t_Y denotes unit tax on the good produced in the industry and t_E is the emission tax. Furthermore an enforcement agency (i.e. Ministry of Environment or environmental protection agency (EPA) as it is called in most of the literature) sets the audit probability α_R (α_U) which is the probability that a firm is discovered underreporting (unreporting) its emission level. If the firm R (the firm U) is caught to have underreported (unreported) emission it has to pay not only the tax on the unreported emission but also a penalty given by the function θ. We assume that this penalty function has the following properties: $\theta(0)=0$, $\theta_d>0$ and $\theta_{dd}>0$.

Let subscripts on a function denote its partial derivatives with respect to the indicated argument; for example, $\theta_d = \partial\theta(d)/\partial d$ and $\theta_{dd} = \partial^2\theta(d)/\partial d^2$ where $d = e^R(q^R,x^R)-z$. Then, within the specifications of the model environment, the first order conditions (FOCs) can be written as:

$$\frac{\partial \Pi^R}{\partial q^R} = -2bq^R + a - bq^U - t_Y - \varphi^R q^R - \alpha_R[(t_E+\theta_d)\delta^R] = 0 \tag{2}$$

[5] For simplicity it is assumed that the firm R does not evade income tax and the firm U does not give any tax on its income. It is evident that the firm R may also under report its income, but we do not intend to tackle this specific case.

$$\frac{\partial \Pi^R}{\partial x^R} = -\phi x^R + \alpha_R(t_E + \theta_d) = 0 \tag{3}$$

$$\frac{\partial \Pi^R}{\partial z} = -t_E + \alpha_R(t_E + \theta_d) = 0 \tag{4}$$

And a little algebra leads to:

$$q^R = \frac{a - bq^U - t_Y - \alpha_R[(t_E + \theta_d)\delta^R]}{2b + \varphi^R} \tag{5}$$

$$x^R = \frac{\alpha_R(t_E + \theta_d)}{\phi} \tag{6}$$

$$t_E = \frac{\alpha_R \theta_d}{1 - \alpha_R} \tag{7}$$

On the other hand, since the firm U has only one control variable, q^U, the maximization problem that it would face can be written as follows:

$$\begin{aligned} Max\Pi^U &= p(q^R + q^U)q^U - c^U(q^U) - \alpha_U[t_E e^U(q^U) + \theta(e^U(q^U))] \\ &\quad -\beta[t_Y q^U + \psi(q^U)] \end{aligned} \tag{8}$$

Similarly β denotes the audit probability that another regulatory authority (i.e. Ministry of Finance) determines aiming at limiting the tax evasion. In other words, with the probability of β, the firm U would be discovered having unreported taxable income and pay the tax and the penalty on its income. Again we assume that the penalty takes the form $\psi(q^U)$ with $\psi_{q^U} > 0$. Note that also we have naturally $\alpha_U, \alpha_R, \beta \in [0,1]$. Next we derive the FOC with respect to q^U:

$$\frac{\partial \Pi^R}{\partial q^U} = -2bq^U + a - bq^R - \varphi^U q^U - \alpha_U[(t_E + \theta_{q^U})\delta^U] - \beta(t_Y + \psi_{q^U}) = 0 \tag{9}$$

which gives finally

$$q^U = \frac{a - bq^R - \alpha_U[(t_E + \theta_{q^U})\delta^U] - \beta(t_Y + \psi_{q^U})}{2b + \varphi^U} \tag{10}$$

Solving simultaneously Eqs. (1) and (2) yields the following proposition which establishes the optimal behavior of the firms R and U which can also be defined as the conditions that the Cournot-Nash equilibrium (henceforth CNE) satisfies.[6]

Proposition 1 For given audit probabilities α_R, α_U and β, tax rates t_Y and t_E, penalty functions, θ and ψ, the optimal production decisions (q^{R*}, q^{U*}) for the firms R and U with parameters (δ^R, φ^R and δ^U, φ^U) are

$$q^{R*} = \frac{[a - t_Y - \alpha_R((t_E + \theta_d)\delta^R)](2b + \varphi^U) + b[-a + \alpha_U((t_E + \theta_{q^U})\delta^U) + \beta(t_Y + \psi_{q^U})]}{(2b + \varphi^U)(2b + \varphi^R) - b^2} \tag{11}$$

$$q^{U*} = \frac{[a - \alpha_U((t_E + \theta_{q^U})\delta^U) - \beta(t_Y + \psi_{q^U})](2b + \varphi^R) + b[-a + \alpha_R((t_E + \theta_d)\delta^R) + t_Y]}{(2b + \varphi^U)(2b + \varphi^R) - b^2} \tag{12}$$

■

[6] For a comprehensive overview of the history of game theory, with a particular focus on the CNE see Myerson (1999).

While after some tedious but simple algebra, using Eqs. (11) and (12) we can calculate Π^{R*}, Π^{U*} and p^*. However, to conserve space, we do not provide further details on such analysis as the main focus of the paper is to examine the effect of environmental regulation on both recorded and unrecorded economic activities.

Eqs. (5) and (10) are called best-response functions which are illustrated in Fig. 1.

[Figure 1 here]

The intersection points of best-response functions and q^R and q^U axis given in Fig. 1, A, B, C and D have parametric values of $\frac{a-t_Y-\alpha_R[(t_E+\theta_d)\delta^R]}{b}$, $\frac{a-\alpha_U[(t_E+\theta_{q^U})\delta^U]-\beta(t_Y+\psi_{q^U})}{2b+\varphi^U}$, $\frac{a-t_Y-\alpha_R[(t_E+\theta_d)\delta^R]}{2b+\varphi^R}$ and $\frac{a-\alpha_U[(t_E+\theta_{q^U})\delta^U]-\beta(t_Y+\psi_{q^U})}{b}$ respectively.

Now, we shall give the following lemma requiring in the proof of some propositions made in the remaining of the paper.

Lemma 1 As there is no information sharing between environmental and finance regulatory authorities, the environmental regulation is conducted using an enforcement mechanism which utilizes an exogenous audit probability for all types of firms whether they have recorded or unrecorded economic activities. This means that there is no reason to have $\alpha_U \neq \alpha_R$.

Proposition 2 If one supposes that the firm R and the firm U have symmetric cost functions, that is, $\varphi^R = \varphi^U$, then the stability condition of the CNE given in Eqs. (11) and (12) requires that $\psi_{q^U q^U} > 0$.

Proof. We may give the intuition behind the proof of this proposition as follows. Even though the CNE results from a static game, if one considers a dynamic game where, in each period, the firm R (the firm U) determines its production level taking into account the production level of the firm U (the firm R) in the previous period, in order to converge step by step to the intersection point $(E(q^{R*}, q^{U*}))$ given in Fig. 1, the slope of the best-response function of the firm R should be higher than that of the firm U. Hence, the following inequality should hold:

$$\frac{-b - \alpha_U {\delta^U}^2 \theta_{dd} - \beta \psi_{q^U q^U}}{2b + \varphi^U} < \frac{-b - \alpha_R {\delta^U}^2 \theta_{dd}}{2b + \varphi^R} \tag{13}$$

from which, applying Lemma 1 for $\alpha_U = \alpha_R$, we can see that the penalty function which is assumed to be increasing function of q^U should also be convex. More formally, that is, we have $\psi_{q^U q^U} > 0$. ■

We close this section by a further observation on the variations of q^{R*} and q^{U*} resulting from a change in the model parameters. The next propositions consider the effects of both environmental tax and audit probability on the size of unrecorded economy.

Lemma 2 In an economy the size of unrecorded economy can be defined and measured simply by $\frac{q^U}{q^R + q^U}$. Thus, using Eqs. (11) and (12) the size of unrecorded economy at the CNE can be calculated analytically from the equation below:

$$\frac{q^{U*}}{q^{R*} + q^{U*}} = \frac{X(2b + \varphi^R) + bY}{X(b + \varphi^R) - (b + \varphi^U)Y} \tag{14}$$

where $X = a - \alpha_U((t_E + \theta_{q^U})\delta^U) - \beta(t_Y + \psi_{q^U})$ and $Y = -a + \alpha_R((t_E + \theta_d)\delta^R) + t_Y$

Proposition 3 Suppose a rise in the environmental tax rate, then a sufficient condition for an increase of the size of unrecorded economy is given by: $\frac{\delta^R}{\delta^U} \geq \frac{2b + \varphi^R}{b}$

Proof. From Eq. (14) it can be seen that $X_{t_E} < 0$ and $Y_{t_E} > 0$. Thus the variation of the denominator is negative if the environmental enforcement agency increases the tax rate t_E. In this case, if the nominator does not decrease the size of unrecorded economy will *a fortiori* increase. This yields after some algebra and rearrangements $b\alpha_R\delta^R \geq \alpha_R(2b+\varphi^R)\delta^U$. It also follows from our Lemma 1: $\frac{\delta^R}{\delta^U} \geq \frac{2b+\varphi^R}{b}$.[7] ■

Before we deal with the policy issues, let us make it clear that this result implies that if the production of the firm R is environmentally less efficient with respect to that of the firm U, that is, higher emission coefficient ($\delta^R > \delta^U$) and if the regulatory authority decides to decrease the emissions by increasing environmental tax, this will have as a consequence a larger extent of the unrecorded economy. Although it is more plausible for the firm R (which explicitly recognizes that it faces environmental regulation) to be less polluting, Proposition 3 shows that if it is not the case, environmental regulation can raise the proportion of total output produced by the firm U. Meanwhile, we may also establish the following proposition.

Proposition 4 The optimal audit probability for the pollutant emissions is $\alpha_R = 1 - \frac{\theta_d}{\phi x^R}$.

Proof. First, use Eq. (6) to write $\alpha_R = \frac{\phi x^R}{t_E+\theta_d}$. Then substitute t_E by its value given in Eq. (7). Finally after some elementary manipulations get: $\alpha_R = 1 - \frac{\theta_d}{\phi x^R}$ ■

Corollary 1 An increase in the audit probability may have an adverse effect on the recorded economic activities.

[7] Although this result with Proposition 2 is theoretically of some interest, it seems very unlikely that it is consistent with the problem studied here. The intuition behind this is that in the context of a dynamic game each firm should observe perfectly reaction of its rival (namely the production level). However, by definition, the production level of the firm U cannot be observed directly.

Proof Following the same procedure given in the proof of Proposition 3 and again by using Lemma 1, one can prove that an increase in α_R may increase the size of unrecorded economy if the following sufficient condition is satisfied: $t_E = \frac{\theta_d b\delta^R + \theta_{q^U}\delta^U(2b+\varphi^R)}{b\delta^R - \delta^U(2b+\varphi^R)}$. ■

Combining Proposition 3 with the foregoing results appears to provide a theoretical ground for the study by Karanfil (2008) who claims that "if environmental taxes are used without reducing the overall economic costs associated with the tax system, no double dividend occurs, hence the shift in tax burden, which is certainly the driving source behind the unrecorded economy, may increase the size of unrecorded economy". As a matter of fact, in what follows, we want to go further than this proposition and provide a threshold rate of environmental tax, exceeding which may lead to an increase of the size of unrecorded economy. Before we deal with this issue, for an extension of this model we introduce a quantity competition game à la Stackelberg, where the firm R is the leader and the firm U the follower. If this assumption seems unwarranted, it is, though not *ad absurdum* since in developing countries counterfeit production represents an important part of the unrecorded economy.[8]

Proposition 5 In the Stackelberg equilibrium if only $\frac{\delta^R}{\delta^U} \geq \frac{b}{2b+\varphi^U}$, then an increase in the environmental tax impedes recorded economic activities.

Proof. See the non-cooperative policy game in Appendix A. ■

If one compares this result with that derived from Proposition 3, one observes that $\frac{2b+\varphi^R}{b} > \frac{b}{2b+\varphi^U}$. This means simply that the minimum value of the relative environmental efficiency ratio $\frac{\delta^R}{\delta^U}$ to have a rise in the extent of unrecorded economy after a

[8] Neylor (1996) provides a nice perspective on the evolution of the *modern* underground economy in which flourish activities like smuggling and counterfeiting.

shift in the environmental tax rate at the CNE is greater than that in the Stackelberg game to have a negative impact of an increase in environmental tax on the recorded activities.

The purpose of the next section is, in addition to an assessment of the effects of a rise in environmental tax on the extent of unrecorded economic activities, to re-examine firms' behaviors subject to a coordinated audit policy between environmental and fiscal authorities who determine endogenously a unique probability-to-audit function.

4 Cooperative policy

In this section we consider a remarkably different enforcement mechanism design which can be outlined as the following: (1) The information is symmetric between the environmental and fiscal enforcement authorities. This assumption can also be interpreted as if there exists only one enforcement agency which audits firms on their both emissions and productions. As a result, if a firm caught to be underreporting its production, *at the same time*, it can also be discovered underreporting its emissions. (2) The audit probability is no more exogenous. The environmental and fiscal enforcement authorities (or the *general* regulator) determine(s) a probability-to-audit function ($\mu(.)$) based on the information available from the recorded economic activities, which are q^R and z. We suppose that only the reported production is used for this purpose, that is, we have $\mu(q^R)$. (3) The form of the probability-to-audit function, whether increasing or decreasing with respect to q^R may be a feature of importance in the enforcement mechanism design. Therefore, some time series analysis have been performed in order to estimate the relationship between q^R and the size of unrecorded economy. The intuition behind our approach is that if, for instance, the size of unrecorded economy increases in a period of *recorded* economic growth, then having this information, regulatory authorities may increase the audit frequency on both income and emission declarations, that is higher $\mu(q^R)$. We present our data, methodology, and the empirical results in Appendix B. The tests carried out in Appendix B show clearly that there is a long-run equilibrium relationship (cointegration) between q^R and the size of unrecorded economy and that the higher the *recorded* production (q^R), the greater the size of unrecorded economy (i.e. q^U/Q). In consequence, since q^R can be observed, assuming that the *general* regulator has such an information, the probability-to-audit function *should* be an increasing function of q^R to combat unrecorded economy. More formally we have $\mu_{q^R} > 0$.

In this cooperative policy case the maximization problems faced by each firm (firm R and firm U) are transformed from Eqs. (1) and (8) to the following Eqs. (15) and (16) respectively.

$$\begin{aligned} Max\Pi^R &= [p(q^R + q^U) - t_Y]q^R - c^R(q^R, x^R) - t_E z \\ &\quad -\mu(q^R)[t_E(e^R(q^R, x^R) - z) + \theta(e^R(q^R, x^R) - z)] \end{aligned} \tag{15}$$

$$\begin{aligned} Max\Pi^U &= p(q^R + q^U)q^U - c^U(q^U) - \mu(q^R)[t_E e^U(q^U) + \theta(e^U(q^U)) \\ &\quad + t_Y q^U + \psi(q^U)] \end{aligned} \tag{16}$$

Proposition 6 Following the same steps as in the non-cooperative policy case, (see Eqs. (2), (5), (9) and (10)) we arrive at the following expressions for the optimal production decisions.

$$q^{R*} = \frac{[a - t_Y - \mu(q^{R*})((t_E + \theta_d)\delta^R) + \mu_{q^R}(t_E(x^R + z) + \theta(d))](2b + \varphi^U)}{(2b + \varphi^U)(2b + \varphi^R + \mu_{q^{R*}} t_E \delta^R) - b^2} + \frac{b[-a + \mu(q^{R*})((t_E + \theta_{q^U})\delta^U + t_Y + \psi_{q^U})]}{(2b + \varphi^U)(2b + \varphi^R + \mu_{q^{R*}} t_E \delta^R) - b^2} \quad (17)$$

$$q^{U*} = \frac{[a - \mu(q^{R*})((t_E + \theta_d)\delta^U + t_Y + \psi_{q^U})](2b + \varphi^R + \mu_{q^{R*}} t_E \delta^R)}{(2b + \varphi^U)(2b + \varphi^R + \mu_{q^{R*}} t_E \delta^R) - b^2} + \frac{b[-a + t_Y + \mu(q^{R*})((t_E + \theta_{q^{U*}})\delta^R) - \mu_{q^{R*}}(t_E(x^R + z) + \theta(d))]}{(2b + \varphi^U)(2b + \varphi^R + \mu_{q^{R*}} t_E \delta^R) - b^2} \quad (18)$$

which are the production levels at the CNE. ■

On the other hand there would be two more FOCs obtained by differentiating Eq. (15) with respect to x^R and z, that is $\frac{\partial \Pi^R}{\partial x^R} = 0$ and $\frac{\partial \Pi^R}{\partial z} = 0$ which give finally:[9]

$$x^R = \frac{\mu(q^R)(t_E + \theta_d)}{\phi} \quad (19)$$

$$t_E = \frac{\mu(q^R)\theta_d}{1 - \alpha_R} \quad (20)$$

Lemma 3 In the case of a cooperative enforcement policy, using Eqs. (17) and (18)

[9] Not surprisingly, both Eqs. (6) and (19) imply that the higher the audit probability the higher the abatement effort. This fact is also supported by some empirical studies. For example, for the case of Japanese manufacturing industries, Hamamoto (2006) found a significant positive relationship between the stringency of environmental regulations and the R&D expenditures.

the size of unrecorded economy at the CNE can be defined by the following identity:

$$\frac{q^{U*}}{q^{R*}+q^{U*}} = \frac{V(2b+\varphi^R+\mu_{q^{R*}}t_E\delta^R)+Wb}{V(b+\varphi^R+\mu_{q^{R*}}t_E\delta^R)-W(b+\varphi^U)} \tag{21}$$

where $V = a-\mu(q^{R*})((t_E+\theta_d)\delta^U+t_Y+\psi_{q^U})$ and $W = -a+t_Y+\mu(q^{R*})((t_E+\theta_{q^{U*}})\delta^R)-\mu_{q^{R*}}(t_E(x^R+z)+\theta(d))$

Proposition 7 Suppose now, as in Proposition 3, that the regulatory authority decides to increase the environmental tax rate, then the extent of the unrecorded economy may be larger if:

$$t_E \geq \frac{\mu(q^R)[(\delta^U)(2b+\varphi^R)-b\delta^R]+\mu_{q^R}[\delta^R(-a+\mu(q^R)(\theta_d\delta^U+t_Y+\psi_{q^U}))+b(x^R+z)]}{2\mu_{q^R}\delta^R\mu(q^R)\delta^U} \tag{22}$$

Proof. From Eq. (14) it can be seen that $V_{t_E}<0$. Besides, from the empirical study as we have concluded that $\mu_{q^R}>0$, we find that $W_{t_E}>0$. As a result, a rise of the environmental tax rate t_E will decrease the denominator of the Eq. (21). In the present case, taking the first derivative of the nominator with respect to t_E yields after some rearrangements the inequality given in Eq. (22). ■

The important point to bear in mind here is that the right hand side of Eq. (22) (which is strictly positive) may be called as the "non-accelerating unrecorded activity rate of environmental tax" (henceforth NAUARET). If the enforcement agency choses a tax rate higher than the NAUARET then the size of unrecorded economy would likely be greater.

To close this section we suppose, as in the previous section, that market competition is characterized by a Stackelberg game instead of a Cournot game.

Proposition 8 If one solves the Stackelberg model, one finds that an increase in the environmental tax rate may have negative impact on the recorded activities if the following is satisfied:

$$\frac{\delta^R(2b+\varphi^U) - b\delta^U}{b\delta^U + (x+z)(2b+\varphi^U)} \geq \frac{\mu_{q^R}}{\mu(q^R)} \tag{23}$$

Proof. See the cooperative policy game in Appendix A. ■

Proposition 8 shows that in the Stackelberg framework, if the growth rate of the audit probability with respect to the recorded economic activities is smaller than a certain level (the threshold given in Eq. (23)), then a rise in the environmental tax may impede recorded economic activities. Here comes the importance of the forme of the probability-to-audit function.

5 Conclusion and additional remarks

The present paper started out from the observation that although it varies across different countries, the size of unrecorded economy is very large in developing countries. Thus, in both theoretical and applied fields new models are needed that can better capture the effects of fiscal and environmental polices on the overall economy including both recorded and unrecorded activities. The paper has employed a duopolistic competition model where behaviors of two representative firms (firm R and firm U) subject to environmental and fiscal regulation are analyzed in two different cases with respect to the existence of cooperation between environmental and fiscal regulatory authorities. Besides, two types of audit probability are considered: in the first case the probability is exogenous while it is a function of recorded economic activities in the second case. The form of this probability-to-audit function has been experimentally investigated using yearly time series data for Turkey.

In our view, the model specified in this way may be more realistic and structurally correct. In consequence the representation identified in the present paper may be very useful in assessing possible effects of different environmental regulation schemes on firm behavior.

The results of this paper can be summarized by third points. First, if the firm R is environmentally less efficient then the firm U and if the environmental enforcement agency audits the emissions randomly, then a shift in the environmental tax rate may increase the size of unrecorded economy. Second, in the periods of economic growth the regulatory authority should increase its audit effort to combat unrecorded economy. This holds at least for the case of Turkey. Last, there exists a threshold level for the environmental tax that we called non-accelerating unrecorded activity rate of environmental tax (NAUARET) above which the extent of the unrecorded economy may be larger due to an increase in the environmental tax rate.

Finally we point out that this study provides two main directions for future research: the theoretical one is to include unrecorded economy in the existing micro and macro economic models while the empirical one consists of an assessment of the long-run relationship between the size of unrecorded economy and recorded economic growth for developing countries, which will considerably increase our understanding of the environmental regulation-unrecorded economy nexus.

Appendix A. Stackelberg game

We consider a Stackelberg game in which the firm R moves first and then the firm U chooses its quantity to produce taking as given the production level of the firm R.

Non-cooperative policy game

The firm R has the following maximization problem:

$$\begin{aligned} Max\Pi^R &= [p(q^R + q^U(q^R)) - t_Y]q^R - c^R(q^R, x^R) - t_E z \\ &\quad -\alpha_R[t_E(e^R(q^R, x^R) - z) + \theta(e^R(q^R, x^R) - z)] \end{aligned} \quad (24)$$

where $q^U(q^R)$ is substituted by the best response function of the firm U given in Eq. (10). Then, the maximum of Π^R with respect to q^R is found from the FOC, that is, $\frac{\partial \Pi^R}{\partial q^R} = 0$:

$$q^{R*} = \frac{(\varphi^U + b)a - (2b + \varphi^U)t_Y + (t_E + \theta_d)[b\alpha_U\delta^U - \alpha_R\delta^R(2b + \varphi^U)] + b\beta(t_Y + \psi_{q^U})}{\varphi^R(2b + \varphi^U) + 2b(b + \varphi^U)} \quad (25)$$

Using Eq. (25) to replace q^R in Eq. (10) gives after some tedious algebraic calculations the optimal production level of the firm U:

$$\begin{aligned} q^{U*} &= \frac{(\varphi^U + b)[ab - 2b(\alpha_U(t_E + \theta_{q^U})\delta^U + \beta(t_Y + \psi^{q^U}))] + (2b + \varphi^U)[\varphi^R(a - \alpha_U(t_E + \theta_{q^U})\delta^U)]}{(2b + \varphi^U)[\varphi^R(2b + \varphi^U) + 2b(b + \varphi^U)]} \\ &\quad - \frac{(\varphi^R + b^2)\beta(t_Y + \psi^{q^U}) + b(t_E + \theta_{q^U})(b + \alpha_U\delta^U - \alpha_R\delta^R}{(2b + \varphi^U)[\varphi^R(2b + \varphi^U) + 2b(b + \varphi^U)]} \end{aligned} \quad (26)$$

Now for the proof of Proposition 5, first we use Lemma 1 to write $\alpha = \alpha_R = \alpha_U$ in Eq. (25), then calculate $\frac{\partial q^R}{\partial t_E}$ which gives finally:

$$\frac{\partial q^R}{\partial t_E} = -\alpha \frac{-b\delta^U + 2\delta^R b + \delta^R \varphi^U}{2\varphi^R b + \varphi^{R+U} + 2b\varphi^U + 2b^2}$$

We arrive at a conclusion that a shift in environmental tax may harm recorded economic activities ($\frac{\partial q^R}{\partial t_E} < 0$) if the firm R environmentally less efficient then the firm U (i.e. to produce same quantity the emission level of firm R is higher than that of firm U). More exactly the relationship between these two parameters should be as given below:

$$\frac{\delta^R}{\delta^U} > \frac{b}{2b + \varphi^U}$$

Cooperative policy game

The maximization problem that the firm R faces is now given by:

$$\begin{aligned} Max\Pi^R &= [p(q^R + q^U(q^R)) - t_Y]q^R - c^R(q^R, x^R) - t_E z \\ &\quad -\mu(q^R)[t_E(e^R(q^R, x^R) - z) + \theta(e^R(q^R, x^R) - z)] \end{aligned} \tag{27}$$

where $q^U(q^R) = \frac{a - bq^R - \mu(q^R)[(t_E + \theta_d)\delta^U + (t_Y + \psi_{q^U})]}{2b + \varphi^U}$. The FOC from $\frac{\partial \Pi^R}{\partial q^R} = 0$ gives

$$\begin{aligned} q^{R*} &= \frac{(\varphi^U + b)a - (2b + \varphi^U)t_Y + \mu(q^R)[(t_E + \theta_d)[b\delta^U - \delta^R(2b + \varphi^U)] + b(t_Y + \psi_{q^U})]}{(\varphi^R + \mu_{q^R} t_E \delta^R)(2b + \varphi^U) + 2b(b + \varphi^U)} \\ &\quad + \frac{\mu_{q^R}[b[(t_E + \theta_d)\delta^U + t_Y + \psi_{q^U}] - (2b + \varphi^U)(t_E(-x - z) + \theta(d))]}{(\varphi^R + \mu_{q^R} t_E \delta^R)(2b + \varphi^U) + 2b(b + \varphi^U)} \end{aligned} \tag{28}$$

Suppose there is a rise in the environmental tax, as the denominator of q^{R*} given in Eq. (28) increases, the sufficient condition for the proof of Proposition 8 can be deducted if the nominator decreases or remains stable. Solving this condition yields the threshold level stated in Eq. (23).

Appendix B. Empirical results

In this Appendix we determine the form of the probability-to-audit function $\mu(q^R)$ from standard time series analysis based on the Turkish data. For this purpose we use the annual data for recorded economy (henceforth RE) taken from the Central Bank of the Republic of Turkey. Note that in the model environment $RE = q^R$. The data used for the size of unrecorded economy (henceforth SUE, that is q^U/Q) is the product of the estimations of unrecorded economy based on the environmental method from Karanfil and Ozkaya (2007). In order to check the robustness of the results both Savasan's (2003) and Schneider and Savasan's (2007) estimations of unrecorded economy are also used. All variables are denoted in real terms and converted into natural logarithms.

Fist of all, time series properties are checked by performing the augmented Dickey Fuller (ADF; Dickey and Fuller, 1981) and the Phillips and Perron (PP; Phillips and Perron, 1988) unit root tests based on the following model:

$$\Delta RE_t = \gamma_0 + \rho t + \gamma_1 RE_{t-1} + \sum_{i=1}^{k} \lambda_i \Delta RE_{t-i} + u_t \tag{29}$$

where RE is the variable to be tested, t is the trend variable, Δ is the first-difference operator and u_t is Gaussian white noise.

[Table 2 here]

In both the cointegration technique developed by Engle and Granger (1987) and Johansen and Juselius' (1990) maximum likelihood procedure in order to establish a long-run equilibrium relationship between two or more variables, the variables should be all non-stationary and integrated of the same order. According to the unit root results reported in Table 2, we can conclude that the variables SUE and RE are both of them non-stationary and integrated of order 1, that is, $I(1)$. Now, we can proceed to the next step which is to perform a cointegration test employing both the maximum eigenvalue and trace statistics.[10]

[Table 3 here]

The results from Table 3 suggest that with 95% confidence level, the variables SUE and RE are cointegrated which means that a long-run equilibrium relationship can be established between the variables involved. Furthermore from the estimated cointegrating vector the following equation can be written.

$$SUE = 0.954128RE - 7.903026$$

We see here clearly that SUE increases when there is a growth in the recorded economic activities.

[10] The aim of this Appendix is not so much to discuss the methodological issues relating to both unit root and cointegration tests. The reader is referred to Hamilton (1994, chapters 11 and 19) for a further information.

We followed the Engle and Granger (1987) two-step procedure to assess the robustness of the cointegration test results. On the other hand, to be sure that the results are not biased due to the choice of data, we have also done work by using the data for unrecorded economy from Savasan (2003) and Schneider and Savasan (2007) and have reached very similar results; the relevant variables are found to be cointegrated and the resulting cointegration equation is $SUE = 0.543394RE$, which establishes, once again, a positive linkage between SUE and RE. These additional results are available upon request from the author.

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Table 1: The average size of the unrecorded economy in developed and less developed countries

Countries/Continents		Size as % of GNP
Developed	OECD countries	12
Transition	Former Soviet Union	25
	Middle and Eastern Europe	20
Developing	Africa	44
	Latin America	39
	Asia	35

Source: Gerxhani (2004: 268, Table 1).

Table 2: Results of unit root tests

Variable	Augmented Dickey-Fuller (ADF)		Phillips-Perron (PP)	
	Levels	First differences	Levels	First differences
SUE	-2.924	-4.772	-3.034	-4.869
RE	-2.536	-6.239	-2.675	-6.265
Critical values				
1%	-4.334	-3.723	-4.334	-3.723
5%	-3.580	-2.989	-3.580	-2.989
10%	-3.228	-2.625	-3.228	-2.625

Table 3: Johansen Test for the number of cointegrating relationships

Eigenvalue	$H_0 : r =$	Trace	L Max	Critical values at 95%	
				Trace	L Max
0.530128	0	27.78722	21.90356	19.96	15.67
0.183628	1	5.883655	5.883655	9.24	9.24

r indicates the number of cointegrating relationships. The critical values for Maximum eigenvalue and trace test statistics are given by Johansen and Juselius (1990).The model specification includes an intercept and no trend in the cointegrating equations.

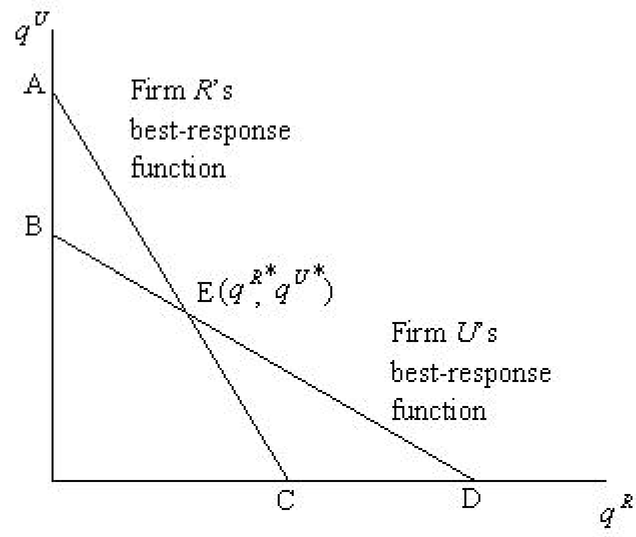

Figure 1: Firms' best-response functions